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Planet temperatures with surface cooling parameterized

Barton Paul Levenson *

631 Montclair Street, Pittsburgh, PA 15217-2808, United States

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Abstract

A semigray (shortwave and longwave) surface temperature model is developed from conditions on Venus, Earth and Mars, where the greenhouse effect is mostly due to carbon dioxide and water vapor. In addition to estimating longwave optical depths, parameterizations are developed for surface cooling due to shortwave absorption in the atmosphere, and for convective (sensible and latent) heat transfer. An approximation to the Clausius–Clapeyron relation provides water–vapor feedback. The resulting iterative algorithm is applied to three "super-Earths" in the Gliese 581 system, including the "Goldilocks" planet g (Vogt et al., 2010). Surprisingly, none of the three appear habitable. One cannot accurately locate a star's habitable zone without data or assumptions about a planet's atmosphere.

Keywords: Planetary atmospheres; Exosolar planets

1. Introduction

One does not always need multilayer atmosphere models with detailed radiation codes to estimate a planet's surface temperature. A gray or semigray model can give estimates close enough for quick comparisons between planets, at negligible cost in computer time. Examples are those of Hart (1978) used for Earth history simulation, Walker et al. (1981) for the Faint Young Sun problem, McKay and Davis (1991) for early Mars, and McKay et al. (1999) for the antigreenhouse effect of haze layers on Titan and early Earth.

Such methods typically ignore aerosols, clouds, and surface cooling mechanisms. A method proposed here parameterizes the latter to account for surface heat losses at the expense of the atmosphere. The effects of aerosols and clouds are subsumed under albedo and the greenhouse effect of water vapor.

2. Radiation model

The model was fitted to Venus, Earth and Mars. These all have surfaces hotter on average than their radiative equilibrium temperatures, mainly from the greenhouse effect of water vapor and carbon dioxide. Table 1 lists parameters of interest for the three worlds.

A planet's climate system absorbs a flux density F:

$$F = (S/4)(1 - A), \tag{1}$$

where S is the local Solar constant and A the bolometric Russell-Bond spherical albedo. This study used Lean's (2000) mean figure for 1951–2000 ($1366\ \mathrm{W\,m^{-2}}$), for Earth's Solar constant. The inverse-square law then yields S for other planets.

Semigray partial optical depths were calculated for carbon dioxide and water vapor by simple power-law parameterizations. This led to an expression for total atmospheric longwave optical depth τ:

$$\tau = 0.025 P_{\mathrm{CO_2}}^{0.53} + 0.277 P_{\mathrm{H_2O}}^{0.3} \tag{2}$$

where $P_{\mathrm{CO_2}}$ and $P_{\mathrm{H_2O}}$ are CO_2 and H_2O partial pressure, respectively (in Pa).

The Eddington–Milne approximation relates T_0 and Te through the gray IR optical depth τ:

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* Tel.: +1 412 422 4555.
E-mail address: levenson1960@gmail.com

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Table 1
Observed terrestrial planet conditions.

Parameter	Venus	Earth	Mars
Surface pressure, Ps (Pa)	9,210,000	101,325	636
Surface temperature, Ts (K)	735.3	288.15	214.0
Solar constant, S (W m^{-2})	2611.0	1366.1	588.45
Bolometric Bond albedo, A	0.750	0.306	0.250
Absorbed flux density, F (W m^{-2})	163	237	110
Equilibrium temperature, Te (K)	232	254	210
Carbon dioxide fraction, X_{CO_2}	0.965	0.000332	0.9532
Water vapor fraction, X_{H_2O}	0.000030	0.003870	0.000210
CO_2 partial pressure, P_{CO_2} (Pa)	8,890,000	33.6	606
H_2O partial pressure, P_{H_2O} (Pa)	300	392	0.13

Sources: Lodders and Fegley (1998), NASA worldwide web planetary fact sheets, calculations by the author from standard formulae.

$$T_0 = Te\left(1 + \frac{3}{4}\tau\right)^{0.25} \tag{3}$$

Substituting Eq. (2) into (3), one can separate out the effect of each gas, finding a surface thermal IR flux density from each:

$$F_{CO_2} = 0.75F\tau_{CO_2} \tag{4a}$$

$$F_{H_2O} = 0.75F\tau_{H_2O} \tag{4b}$$

Subtracting the atmosphere-absorbed flux density from F, we can then define surface shortwave illumination Fsi. The surface energy balance is then:

$$Fs = Fsi + F_{CO_2} + F_{H_2O} - Fc \tag{5}$$

where Fs is flux density radiated by the surface, and convective loss Fc – note the sign – is from sensible and latent heat transfer. The surface temperature follows from inverting the Stefan–Boltzmann radiation law:

$$Ts = (Fs/[\varepsilon\sigma])^{0.25} \tag{6}$$

Here σ is the Stefan–Boltzmann constant. Emissivity ε was taken as 0.95 for Venus and Mars, 0.996 for Earth, after previous estimates in the literature (e.g. Roeckner et al., 2003).

3. Atmospheric absorption

Surface heat loss from sunlight absorbed in the atmosphere was modeled with the Beer–Lambert–Bouguer law. The shortwave flux density each planet's climate system absorbs is $F = 163$, 237 and 110 W m^{-2}, respectively for Venus, Earth and Mars. Mean shortwave illumination at the surface is estimated at $Fsi = 16.8$ W m^{-2} for Venus (Marov and Grinspoon, 1998, p. 298). For Earth, Trenberth et al. (2009) find $Fsi = 161.2$ W m^{-2}.

No good figure is available for Mars. Courtin and Murdin (2001) estimate $Fsi = 102$ W m^{-2}, implying very low visual optical depth ($\tau_v = 0.0755$). On the other hand, Read and Lewis (2004, p. 52) suggest atmospheric shortwave absorption of 45% for "clear" conditions ($\tau_v = 0.6$) and 85% in a major dust storm ($\tau_v = 5$). No mean annual global estimate is given.

For Mars to absorb a higher sunlight proportion than Earth despite 200 times less atmosphere mass seems counterintuitive, but of course Mars has extensive dust storms – and low-altitude dust heats lower layers of atmosphere and thus the surface.

The present model omits clouds and dust. A "clear" Martian atmosphere would presumably absorb less sunlight than Earth's, leading to a τ_v figure closer to Courtin and Murdin's.

For "visible" (shortwave) light, optical depth is:

$$\tau_v = -\ln(I/I_0) \tag{7a}$$

where I and I_0 are shortwave irradiance received and entering the media column, respectively. From Houghton (2002, p. 10), the three-dimensional irradiance can be translated to one-dimensional flux density by removing the dimension of solid angle, i.e., dividing by π steradians. With the same denominator for I and I_0, substitution yields:

$$\tau_v = -\ln(Fsi/F) \tag{7b}$$

for shortwave optical depth to the surface. Given the Fsi figures above, $\tau_v = 2.27$, 0.385 and 0.0755 for Venus, Earth and Mars, respectively. Ozawa and Ohmura (1997) suggest that τ_v should be linearly related to τ. A linear fit to Earth and Venus figures is:

$$\tau_v = 0.354 + 0.0157\tau \tag{8}$$

This gives a figure of $\tau_v = 0.368$ for Mars, intermediate between Courtin and Murdin (2001) and Read and Lewis (2004).

Fsi is then

$$Fsi = F\exp(-\tau_v) \tag{9}$$

4. Convective heat transfer

Lorenz and McKay (2003) parameterize this with an equation of the form:

$$Fc = Fsi\tau/(C + D\tau) \tag{10}$$

where C and D are constants. With constants fitted to Earth's observed convective flux, 97 W m^{-2} (Trenberth et al., 2009), a range of values is possible for C and D. However, it is nearly impossible to fit Venus, Earth and Mars at the same time without unphysical results (negative Fc) for at least one planet. A simpler parameterization was therefore adopted, based on a linear fit between Earth and Mars figures (Fc on Venus is not known). $Fc = 97$ W m^{-2} for Earth (Trenberth et al., 2009) and perhaps 5 W m^{-2} for Mars (Lorenz and McKay, 2003). This gives:

$$Fc = -22.5 + 0.402Fsi\tau \tag{11}$$

This yields realistic figures for all three planets. Of course, if the product $Fsi\ \tau < 55.97$, the result is again unphysical, and $Fc = 0$ should be substituted.

5. Water–vapor feedback

In an atmosphere where water is condensable – i.e., that of any habitable planet – water vapor pressure depends on temperature by the Clausius–Clapeyron ("CC") relation, suitably modified for relative humidity. Then Ts depends on P_{H_2O}, while P_{H_2O} depends on Ts. But since the CC curve is roughly exponential, while Ts depends on the one-fourth power of flux density, one can start with $Ts = Te$ and iterate until the results converge.

The water vapor scheme derives from a parameterization in Hart (1978). The equation used here is:

$$P_{H_2O} = 392 \exp(Q_2[Ts - 288.15]) \tag{12}$$

where Q_2 is a constant equal to $0.0698\ \mathrm{K}^{-1}$. 392 Pa is Earth's present global mean P_{H_2O}, while Earth's $Ts = 288.15$ K in the US Standard Atmosphere (NOAA et al., 1976).

For $Ts > 373.15$ K, P_{H_2O} was set to a fixed 147,896 Pa, the value Eq. (12) gives at the boiling point of water. The model begins by assuming $Ts = Te$ and iterates until the change in Ts falls below 0.001 K.

For Venus and Mars, the fixed observed levels of water vapor were used, since the temperature regimes and major condensable substances on each are different from Earth.

6. Climate sensitivity of the model

Houghton (2004, p. 24) estimates Earth's climate sensitivity to doubled carbon dioxide, ΔT_{2X}, at 1.2 K with no climate feedbacks. The Intergovernmental Panel on Climate Change (IPCC, 2007, p. 9) estimates $\Delta T_{2X} = 2.0$–4.5 K with feedbacks included, with a best estimate ≈ 3 K.

To test this model's climate sensitivity, it was run on a hypothetical planet, "Earth B," with doubled carbon dioxide. For Earth B, Ts = 294.1 K compared to 288.4 K for Earth. This gives a climate sensitivity $\Delta T_{2X} = 5.8$ K, above the IPCC range. The model is perhaps oversensitive to changes in carbon dioxide. On the other hand, the Houghton and IPCC figures are for the relatively short-term Charney sensitivity (Charney et al., 1979). Hansen and Sato (2007) have suggested that long-term climate sensitivity is, in fact, about 6 K.

The model is not precise enough for climate change studies as is. But it is perhaps close enough for quick-and-dirty comparative planetology, since it does give a close figure for Earth and ballpark figures for Venus and Mars.

7. Results for the solar system

Temperature figures from the model were $Ts = 715.8$, 288.4, and 228.0 K for Venus, Earth, and Mars. More detail is given in Table 2. Relative errors were about 2.7%, 0.1%, and 6.5% for Venus, Earth, and Mars, respectively.

It is interesting to apply the model to each planet, varying the planetary mass from that of Mars to Venus to Earth. We then need an expression for the CO_2 partial pressure.

Earth's total atmospheric pressure and P_{CO_2} have varied widely through geological history (cf. Walker, 1977). This is no doubt true for other planets. To estimate pressure figures for a planet with nothing to go on but its mass is risky at best. Nonetheless, some parameterization was needed to continue the analysis.

Table 2
Model radiative parameters for terrestrial planets.

Parameter	Venus	Earth	Mars
CO_2 partial optical depth, τ_{CO_2}	120	0.161	0.746
H_2O partial optical depth, τ_{H_2O}	1.53	1.67	0.150
Total gray IR optical depth, τ^{a}	122	1.83	0.896
"Raw" greenhouse temperature, T_0 (K)	726	305	230
"Raw" greenhouse flux, F_0 (W m^{-2})	14,900	487	151
Visual optical depth, τ_{vis}	2.27	0.383	0.368
Surface illumination, Fsi (W m^{-2})	16.9	162	76.4
Greenhouse, F_{CO_2} (W m^{-2})	14,700	28.6	61.7
Greenhouse, F_{H_2O} (W m^{-2})	188	297	12.4
Convective loss, Fc (W m^{-2})	805	96.4	5.0
Net surface flux, Fs (W m^{-2})	14,100	391	146
Surface temperature, Ts (K)	715.8	288.4	228.0

[a] May show rounding error.

Table 3
Planet Gravities and Adopted Greenhouse Gas Levels.

Planet	g^{a}	Adopted P_{CO_2}	Adopted P_{H_2O}	Ts
Venus$_{\mathrm{M}}$	0.3795	1,570,000.00	53.00	552.6
Venus	0.9033	8,890,000.00	300.00	715.8
Venus$_{\mathrm{E}}$	1.0000	10,900,000.00	368.00	737.4
Earth$_{\mathrm{M}}$	0.3795	4.84	f(Ts)	281.1
Earth$_{\mathrm{V}}$	0.9033	27.40	f(Ts)	287.1
Earth	1.0000	33.60	f(Ts)	288.4
Mars	0.3795	606.00	0.13	228.0
Mars$_{\mathrm{V}}$	0.9033	3430.00	0.74	249.6
Mars$_{\mathrm{E}}$	1.0000	4210.00	0.90	253.1

[a] Relative to Earth.

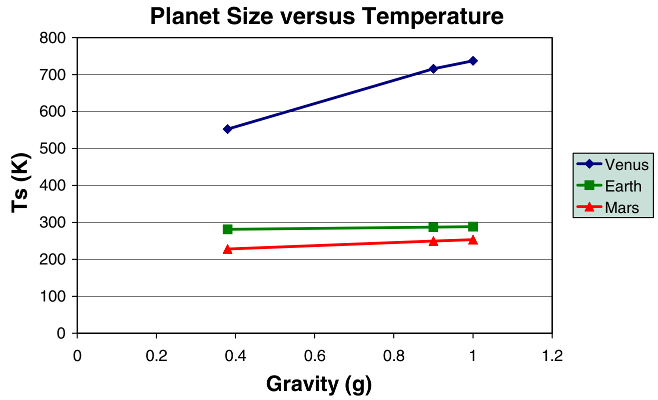

Fig. 1. Planet size versus temperature.

Table 4
Three planets in the Gliese 581 system.

Planet	$M_{\oplus} \sin i$	P (days)	a (AUs)
c	5.4	12.9	0.0700
g	3.2	36.7	0.146
d	7.1	66.8	0.220

Table 5
Radiative equilibrium model of Gliese 581 planets.

Planet	S (W m^{-2})	F (W m^{-2})	Te (K)
c	3624	628.8	325
g	833	144.5	225
d	367	63.7	183

It may be argued that the mass of atmosphere per unit area on a planet is related to the planet's mass. Gravitational compression of the silicate mantle might result in more outgassing, and higher gravity would yield harder (=>higher-temperature) impacts by volatile-bearing planetesimals and comets. P_{CO_2} should therefore be proportionate to gravity.

It should also be proportionate to gravity, again, because more gravity directly forces higher atmospheric pressure. The amount, therefore, may be proportionate to gravity squared:

$$P_{CO_2} = kg^2 \tag{13}$$

where k is relative to the planet's known atmospheric mass at the original planetary mass, and g is relative to Earth. With this assumption, the model was applied to three real and six hypothetical planets, the latter being:

Venus$_M$	Venus with Mars mass
Venus$_E$	Venus with Earth mass
Earth$_M$	Earth with Mars mass
Earth$_V$	Earth with Venus mass
Mars$_V$	Mars with Venus mass
Mars$_E$	Mars with Earth mass

Table 3 shows the results. For Venus and Mars planets, water vapor was fixed proportionate to g^2. Fig. 1 graphs the results.

A smaller Venus is cooler, but not enough to be habitable. Nor is a larger Mars habitable. Earth is habitable at all three sizes. As with smaller amounts of real estate, what matters most is location, location, location.

8. The model applied to three planets of Gliese 581

Since 2005, several exoplanets have been discovered which may be more "Terrestrial" (rocky) than Jovian or Neptunian. The model here was applied to three planets in the Gliese 581 system (Bonfils et al., 2005; Udry et al.,

Table 6
Modeled characteristics of Gliese 581 planets.

Planet	i (°)	Mass (Me)	P_{CO_2} (Pa)	P_{H_2O} (Pa)	τ	Ts (K)
c	1	309.4	25225.2	147896.0	15.232	556.7
	10	31.1	1753.7	147896.0	11.161	515.7
	20	15.8	853.3	147896.0	10.745	511.0
	30	10.8	577.4	147896.0	10.578	509.1
	40	8.4	448.3	147896.0	10.486	508.1
	50	7.0	376.5	147896.0	10.430	507.5
	60	6.2	333.7	147896.0	10.394	507.0
	70	5.7	308.1	147896.0	10.372	506.8
	80	5.5	294.3	147896.0	10.359	506.6
	90	5.4	290.0	147896.0	10.355	506.6
g	1	183.4	13337.6	10607.7	8.308	335.4
	10	18.4	1003.0	46.0	1.848	257.5
	20	9.4	499.4	26.1	1.411	249.4
	30	6.4	342.3	20.6	1.238	245.9
	40	5.0	268.0	18.0	1.144	244.0
	50	4.2	226.4	16.6	1.086	242.8
	60	3.7	201.5	15.7	1.049	242.1
	70	3.4	186.5	15.2	1.026	241.6
	80	3.2	178.4	14.9	1.013	241.3
	90	3.2	175.9	14.8	1.009	241.2
d	1	406.8	35445.7	117.1	7.601	270.8
	10	40.9	2365.9	2.6	1.904	216.3
	20	20.8	1137.2	1.5	1.356	208.7
	30	14.2	764.4	1.2	1.138	205.5
	40	11.0	590.8	1.1	1.019	203.6
	50	9.3	494.7	1.0	0.946	202.5
	60	8.2	437.5	0.9	0.900	201.8
	70	7.6	403.3	0.9	0.871	201.3
	80	7.2	385.0	0.9	0.855	201.0
	90	7.1	379.2	0.9	0.849	200.9

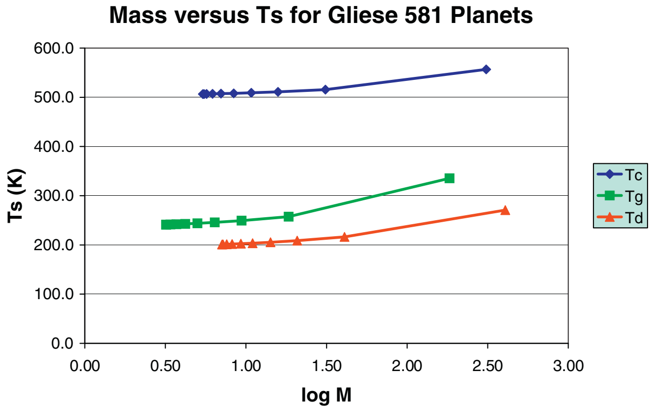

Fig. 2. Mass versus Ts for Gliese 581 planets.

2007; Vogt et al., 2010 – see also Selsis et al., 2007; von Bloh et al., 2007). One is the possibly Earthlike "Goldilocks" planet, Gliese 581 g.

Gliese 581 is a main sequence red dwarf of spectral class M3. Its luminosity is estimated at $L/L_\odot = 0.013$. Table 4 lists characteristics of planets *c*, *g*, and *d*, ordered by increasing semimajor axis.

Unfortunately, the "masses" for these planets in popular reporting–and even sometimes in the professional literature! – are actually $m \sin i$ figures, since the orbital inclinations i are unknown. With low inclination, the actual masses might be Jovian, or even those of brown dwarfs. The analysis was therefore carried out for possible orbital plane inclinations of 0° through 90° in 10° increments. An inclination of 1° was substituted for the 0° figure to avoid infinite values.

On the assumption that the planets may be Earthlike, each was given Earth's albedo, $A = 0.306$ (NASA datum). Since the semimajor axes are known, the "solar" constant for each planet could be calculated. This yields climate flux densities and radiative equilibrium temperatures for each world. Table 5 gives these.

Each world was assigned an atmosphere like that of pre-industrial Earth (X_{CO_2} 280 ppmv). CO_2 partial pressures followed equation 14. To estimate planetary surface gravities, a density-radius relation developed by Dole (1964, pp. 27–30) for planets of Earthlike composition was used. These parameterizations allowed completion of Table 6 data. Fig. 2 plots mass versus temperature for all three Gliese 581 planets.

Surprisingly, it is not likely that any of the planets is habitable. Gliese 581 c is very likely a desert like Venus, while d is frozen. There is a narrow range of inclinations in which Gliese 581 *g* may have liquid–water temperatures, but if so, they are coupled with gravity so extreme it is difficult to visualize higher life evolving. Speculation about Gliese 581 *g* being an Earthlike "Goldilocks" planet may have been premature.

9. Conclusion

The location of a star's habitable zone is usually calculated for an Earthlike planet with a similar atmosphere. If the atmosphere is not as expected, even if of Earthlike composition, the boundaries of the habitable zone can shift considerably.

As yet we know nothing about Gliese 581 *g*'s actual atmosphere, or even if it has one. We cannot yet rule out habitability altogether. This remains a planet worthy of further investigation, even for astrobiologists.

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